

A QCD-Analysis for Radiative Decays of Υ into $f_2(1270)$

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Abstract

We perform a QCD analysis for the radiative decay of a heavy 3S_1 quarkonium into the tensor meson $f_2(1270)$. We make an attempt to separate the nonperturbative effect related to the quarkonium and that related to the tensor meson, the former is represented by NRQCD matrix elements, while the later is parameterized by distribution amplitudes of gluons in the tensor meson at twist-2 level and at twist-3 level. We find that at twist-2 level the helicity λ of the tensor meson can be 0 and 2 and the amplitude with $\lambda = 2$ is suppressed. At twist-3 level the tensor meson can have $\lambda = 1$. A comparison with experiment is made, an agreement of our results with experiment can be found. We also briefly discuss the radiative decay into η and obtain a prediction for $\Upsilon \rightarrow \gamma + \eta$.

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1. Introduction

Decays of heavy quarkonia into light hadrons are forbidden processes by OZI rule, their study will help us to understand how gluons, which are fundamentally dynamical freedoms of QCD, are converted into light hadrons. Radiative decays of quarkonia into a single hadron provide an ideal place to study the conversion of gluons into the light hadron. Because there is only one light hadron in the final state, there is no complication due to complicated interactions between light hadrons, and it is relatively easy to determine the conversion of gluons into the light hadron. In this work, we propose to study the radiative decay into $f_2(1270)$, which is a spin-2 particle and has the mass $m = 1.27\text{GeV}$.

The radiative decay of J/Ψ was observed long time ago, and an analysis of the polarization of f_2 was also performed [1], where the main decay mode $f_2 \rightarrow \pi + \pi$ was used. Recently the decay of Υ was also observed by CLEO [2], the branching ratio was determined.

In the decay the initial state is a heavy quarkonium, say Υ , which can be taken as a bound state of a b - and \bar{b} -quark as an approximation. Then the radiative decay of Υ undergoes as the following: the quarkonium will be annihilated into a real photon and gluons, the gluons will subsequently be converted into the tensor meson f_2 . In the heavy quark limit, the b - and \bar{b} -quark moves with a small velocity v , hence an expansion in v can be employed, nonrelativistic QCD(NRQCD) can be used to describe the nonperturbative effect related to Υ [3]. Also in the limit, the tensor meson f_2 has a large momentum, this enables an expansion in twist to characterize the gluonic conversion into f_2 , the conversion is then described by a set of distribution amplitudes of gluons. The large momentum of f_2 requires that the gluons should be hard, hence the emission of the gluons can be handled by perturbative theory. The above discussion implies that we may factorize the decay amplitude into three parts: the first part consists of matrix elements of NRQCD representing the nonperturbative effect related to Υ , the second part consists of some distribution amplitudes, which are for the gluonic conversion into f_2 , the third part consists of some coefficients, which can be calculated with perturbative theory for the $b\bar{b}$ -pair annihilated into gluons and a real photon. This work is an attempt to obtain a factorized form for the decay amplitude at tree level, in which we analyse the gluon conversion up to twist-3 level and give the definition of the corresponding distribution amplitudes. At loop levels, if the factorization still holds, the perturbative coefficients should be free from infrared singularities, and all nonperturbative effects should be contained in the NRQCD matrix elements and in the distribution amplitudes. To warranty this one needs to prove the factorization at loop levels, which is beyond the current work.

The radiative decay was studied in the framework of a nonrelativistic quark model [4], in which, not only a nonrelativistic description is employed for the initial quarkonium, but also for the tensor meson. The later is not well justified. Instead of this we use a set of gluonic distribution amplitudes for f_2 , which are relativistic, gauge invariant, and universal. Hence once information of them is extracted from one experiment, it can also be used for predictions for other experiments. For example, recently hard exclusive production of f_2 was studied [5], in which the same distribution amplitudes appeared. In this work we will use our results for the two decays: $\Upsilon \rightarrow \gamma + f_2$ and $J/\Psi \rightarrow \gamma + f_2$, and an agreement between

experiment and our results can be reached under certain conditions.

Part of our results may also be used for the radiative decay into η . It turns out that the decay amplitude is at twist-4 level. Without a complete analysis we can still predict the decay width for $\Upsilon \rightarrow \gamma + \eta$. We will briefly discuss this decay mode.

Our work is organized as follows: In Sect.2 we introduce our notation and perform the analysis at twist-2 level. In sect. 3 we perform the analysis at twist-3 level. In Sect. 4 we make a comparison of our results with experiment and give a brief discussion for the decay into η . Sect.5 is our conclusion.

Throughout of our work we take nonrelativistic normalization for the quarkonium state and for heavy quarks.

2. Notations and results at order of twist-2

We consider the exclusive decay of Υ in its rest-frame:

$$\Upsilon(P) \rightarrow \gamma(q) + f_2(k), \quad (1)$$

where the momenta are given in the brackets. Since f_2 is a spin-2 particle, its polarization is described by a symmetric, trace-less tensor $\varepsilon^{\mu\nu}(k, \lambda)$, where λ is the helicity of f_2 . This tensor can be constructed by introducing polarization vectors ω^μ 's. We take a coordinate system in which f_2 moves in the direction of the z -axis. In this system the vectors take the form

$$\omega^\mu(1) = \frac{1}{\sqrt{2}}(0, 1, i, 0), \quad \omega^\mu(-1) = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \quad \omega^\mu(0) = \frac{1}{m}(|\mathbf{k}|, 0, 0, k^0). \quad (2)$$

With these vectors and with Clebsch-Gordan coefficients one can construct $\varepsilon^{\mu\nu}(k, \lambda)$ for $\lambda = 0, \pm 1, \pm 2$. At the leading order of QED the S -matrix element for the decay is

$$\langle \gamma f_2(\lambda) | S | \Upsilon \rangle = -ieQ_b \varepsilon^{*\mu} \cdot \int d^4 z e^{iq \cdot z} \langle f_2(\lambda) | \bar{b}(z) \gamma_\mu b(z) | \Upsilon \rangle, \quad (3)$$

where Q_b is the electric charge of b -quark in unit of e , $b(x)$ is the Dirac field for b -quark, $\varepsilon^{*\mu}$ is the polarization vector of the photon and λ is the helicity of f_2 . In Eq.(3) we only take the part of b -quark in the electric current into account. If two gluons are emitted by the b - or \bar{b} -quark, we obtain the corresponding contribution to the S -matrix element

$$w_2 = -i \frac{1}{2} e Q_b g_s^2 \varepsilon^{*\mu} \int d^4 x d^4 y d^4 z e^{iq \cdot z} \langle f_2(\lambda) | T [\bar{b}(x) \gamma \cdot G(x) b(x) \bar{b}(y) \gamma \cdot G(y) b(y) \bar{b}(z) \gamma^\mu b(z)] | \Upsilon \rangle, \quad (4)$$

where $G(x)$ is the gluon field. Using Wick-theorem we can calculate the T -ordered product and we only keep those terms in which one b -field and one \bar{b} -field remain uncontracted. Then the matrix element takes a complicated form and can be written in a short notation:

$$w_2 = -i \frac{1}{2} e Q_b g_s^2 \varepsilon^{*\mu} \int d^4 x d^4 y d^4 z d^4 x_1 d^4 y_1 e^{iq \cdot z} \langle f_2(\lambda) | G_\mu^a(x) G_\nu^b(y) | 0 \rangle \langle 0 | \bar{b}_j(x_1) b_i(y_1) | \Upsilon \rangle \cdot M_{ji}^{\mu\nu\rho, ab}(x, y, x_1, y_1, z), \quad (5)$$

where $M_{ji}^{\mu\nu\rho,ab}(x, y, x_1, y_1, z)$ is a known function, i and j stand for Dirac- and color indices, a and b is the color of gluon field. The above equation can be generalized to emission of arbitrary number n of gluons, the corresponding contribution is w_n , then the S -matrix element is the sum

$$\langle \gamma f_2(\lambda) | S | \Upsilon \rangle = \sum_n w_n. \quad (6)$$

In each contribution there is the same matrix element $\langle 0 | \bar{b}_j(x) b_i(y) | \Upsilon \rangle$. For this matrix element the expansion in v can be now performed, the result is:

$$\langle 0 | \bar{b}_j(x) b_i(y) | \Upsilon \rangle = -\frac{1}{6} (P_+ \gamma^\ell P_-)_{ij} \langle 0 | \chi^\dagger \sigma^\ell \psi | \Upsilon \rangle e^{-ip \cdot (x+y)} + \mathcal{O}(v^2), \quad (7)$$

where $\chi^\dagger(\psi)$ is the NRQCD field for $\bar{b}(b)$ quark and

$$\begin{aligned} P_\pm &= (1 \pm \gamma^0)/2, \\ p^\mu &= (m_b, 0, 0, 0), \end{aligned} \quad (8)$$

where m_b is the pole-mass of the b -quark. The leading order of the matrix element is $\mathcal{O}(v^0)$, we will neglect the contribution from higher orders and the momentum of Υ is then approximated by $2p$. It should be noted that effects at higher order of v can be added with the expansion in Eq.(7). Taking the result in Eq.(7) we can write the S -matrix element as:

$$\begin{aligned} w_2 &= i \frac{1}{24} e Q_b g_s^2 (2\pi)^4 \delta^4(2p - k - q) \varepsilon_\rho^* \langle 0 | \chi^\dagger \sigma^\ell \psi | \Upsilon \rangle \\ &\quad \int \frac{d^4 q_1}{(2\pi)^4} \Gamma_{2\mu\nu}(k, q_1, \lambda) \cdot R_2^{\mu\nu\rho\ell}(p, k, q_1), \end{aligned} \quad (9)$$

with

$$\Gamma_2^{\mu\nu}(k, q_1, \lambda) = \int dx^4 e^{-iq_1 \cdot x} \langle f_2(\lambda) | G^{a,\mu}(x) G^{a,\nu}(0) | 0 \rangle. \quad (10)$$

To obtain Eq.(9) we have used the color-symmetry and the translational covariance. $\Gamma_2^{\mu\nu}(k, q_1)$ contains all nonperturbative effect related to f_2 , while $R_2^{\mu\nu\rho\ell}(p, k, q_1)$ is a perturbative function, whose physical interpretation is that it is the amplitude for a 3S_1 $b\bar{b}$ pair emitting two gluons and a real photon, and the quarks have the same momentum p . The contribution of w_2 may be represented by the diagrams given in Fig. 1. In Fig.2 we give the diagrams corresponding to emission of three gluons, their contributions w_3 will be analysed in the next section. Emissions of more than three gluons will lead to contributions which are at orders higher than those of twist-3, and will not be considered in this work.

To perform the twist expansion it is convenient to introduce the light-cone coordinate system, in which a vector A is given by $A^\nu = (A^+, A^-, A^1, A^2) = (A^+, A^-, A_T)$, the component A^+ and A^- is related to A^0 and A^3 in the usual coordinate system by

$$A^+ = \frac{1}{\sqrt{2}}(A^0 + A^3), \quad A^- = \frac{1}{\sqrt{2}}(A^0 - A^3). \quad (11)$$

In the light-cone coordinate system the momentum k of f_2 takes the form:

$$k^\mu = (k^+, k^-, 0, 0), \quad k^- = \frac{m^2}{2k^+}. \quad (12)$$

In the heavy quark limit, k^+ is very large, while k^- goes to zero. We also define two vectors n and l and a tensor d_T :

$$\begin{aligned} n^\mu &= (0, 1, 0, 0), \quad l^\mu = (1, 0, 0, 0), \\ d_T^{\mu\nu} &= g^{\mu\nu} - n^\mu l^\nu - n^\nu l^\mu. \end{aligned} \quad (13)$$

Different components of a vector can be projected out with these vectors and the tensor:

$$A^+ = n \cdot A, \quad A^- = l \cdot A, \quad A_T^\mu = d_T^{\mu\nu} A_\nu. \quad (14)$$

The light-cone gauge is chosen in our work and is defined by

$$n \cdot G(x) = G^+(x) = 0. \quad (15)$$

In this gauge a set of components of the gluon field strength takes a simple form:

$$G^{+\mu}(x) = \partial^+ G^\mu(x) = \frac{\partial}{\partial x^-} G^\mu(x). \quad (16)$$

The nonperturbative object $\Gamma_2^{\mu\nu}$ characterize the conversion of two gluons into f_2 . The x -dependence of the matrix element $\langle f_2(k, \lambda) | G^{a,\mu}(x) G^{a,\nu}(0) | 0 \rangle$ is controlled by different scales: the x^- -dependence is controlled by k^+ , while the x^+ - and x_T -dependence is controlled by the scale Λ_{QCD} or k^- , which are small in comparison with k^+ . Because of these small scales we can expand the matrix element in x^+ and in x_T . With this expansion we obtain

$$\begin{aligned} \Gamma_2^{\mu\nu}(k, q_1, \lambda) &= (2\pi)^3 \delta(q_1^-) \delta^2(q_{1T}) \cdot \int dx^- e^{-iq_1^+ x^-} \langle f_2(\lambda) | G^{a,\mu}(x^-) G^{a,\nu}(0) | 0 \rangle \\ &\quad + i(2\pi)^3 \delta(q_1^-) \frac{\partial}{\partial q_{1T}^\rho} \delta^2(q_{1T}) \cdot \int dx^- e^{-iq_1^+ x^-} \langle f_2(\lambda) | \partial_T^\rho G^{a,\mu}(x^-) G^{a,\nu}(0) | 0 \rangle \\ &\quad + i(2\pi)^3 \frac{\partial}{\partial q_1^-} \delta(q_1^-) \delta^2(q_{1T}) \cdot \int dx^- e^{-iq_1^+ x^-} \langle f_2(\lambda) | \partial^- G^{a,\mu}(x^-) G^{a,\nu}(0) | 0 \rangle \\ &\quad + \dots, \end{aligned} \quad (17)$$

where we introduced a short notation:

$$G^\mu(x^-) = G^\mu(x^- n). \quad (18)$$

In Eq.(17), the leading twist of the first term is 2, while the leading twist of the second term is 3. By considering Lorentz covariance and varying the vector n [6] one can show that the derivative ∂^- in the third term is related to ∂_T^2 , hence the leading twist of the third term is 4. The \dots stand for contributions which have twist more than 3 and will be neglected. It should be noted that the expansion in Eq.(17) is equivalent to the collinear expansion around $q_1^\mu = (q_1^+, 0, 0, 0)$ for $R_2^{\mu\nu\rho\ell}(p, k, q_1)$.

The leading-twist contribution in the first term is specified by that the indices μ and ν are all transversal indices, i.e., μ or $\nu = 1, 2$. By considering the covariance under Lorentz boost along the z -axis and the invariance of rotations around the z -axis we find at twist-2 for $\lambda = 0$:

$$\Gamma_2^{\mu\nu}(k, q_1, 0) = (2\pi)^4 \delta(q_1^-) \delta^2(q_{1T}) \cdot \frac{1}{2k^+ x_1 (x_1 - 1)} d_T^{\mu\nu} F_0(x_1), \quad (19)$$

where $q_1^+ = x_1 k^+$, $F_0(x_1)$ is the gluonic distribution amplitude for $\lambda = 0$ and is given by

$$F_0(x_1) = \frac{1}{2\pi k^+} \int dx^- e^{-ix_1 k^+ x^-} \langle f_2(0) | G^{a,+\mu}(x^-) G^{a,+}_{\mu}(0) | 0 \rangle \quad (20)$$

It should be noted that

$$\frac{1}{\sqrt{6}} d_T^{\mu\nu} = \varepsilon_T^{*\mu\nu}(0) = \varepsilon_{\mu'\nu'}^*(0) d_T^{\mu'\mu} d_T^{\nu'\nu}. \quad (21)$$

At twist-2 level $\Gamma^{\mu\nu}(k, q_1, \lambda)$ can also be nonzero for $\lambda = \pm 2$. The contribution for $\lambda = 2$ can be written as

$$\Gamma_2^{\mu\nu}(k, q_1, 2) = (2\pi)^4 \delta(q_1^-) \delta^2(q_{1T}) \cdot \frac{1}{k^+ x_1 (x_1 - 1)} \varepsilon^{*\mu\nu}(2) F_2(x_1), \quad (22)$$

where $F_2(x_1)$ is the gluonic distribution amplitude for $\lambda = 2$ and is defined by:

$$F_2(x_1) = \frac{1}{2\pi k^+} \int dx^- e^{-ix_1 k^+ x^-} \langle f_2(2) | G^{a,+\mu}(x^-) G^{a,+\nu}(0) | 0 \rangle \omega_\mu(1) \omega_\nu(1) \quad (23)$$

The polarization vector $\varepsilon^{\mu\nu}(2)$ is given by

$$\varepsilon^{\mu\nu}(2) = \omega^\mu(1) \omega^\nu(1). \quad (24)$$

At twist-2 level f_2 can not have the helicity $\lambda = 1$, because the two gluons are collinear with a relative angular momentum which is zero and they have Bose-symmetry. With these results for $\Gamma^{\mu\nu}(k, q_1, \lambda)$ and with the explicit form of $R_2^{\mu\nu\rho\ell}(p, k, q_1)$ we obtain the S -matrix element with $\lambda = 0$ and with $\lambda = 2$:

$$\begin{aligned} \langle \gamma f_2(0) | S | \Upsilon \rangle &= \frac{i}{6} e Q_b g_s^2 (2\pi)^4 \delta^4(2p - k - q) \varepsilon^{*\ell} \langle 0 | \chi^\dagger \sigma^\ell \psi | \Upsilon \rangle \cdot \frac{1}{m_b^2} \cdot T_0, \\ \langle \gamma f_2(2) | S | \Upsilon \rangle &= \frac{i}{6} e Q_b g_s^2 (2\pi)^4 \delta^4(2p - k - q) (-\varepsilon_\mu^* \omega^{*\mu}(1)) \omega^{*\ell}(1) \langle 0 | \chi^\dagger \sigma^\ell \psi | \Upsilon \rangle \cdot \frac{1}{m_b^2} \cdot T_2, \end{aligned} \quad (25)$$

with

$$\begin{aligned} T_0 &= \int dx_1 \frac{8m_b^2(x_1 - 1) + m^2(1 - 2x_1)}{2x_1(1 - x_1)(4m_b^2(x_1 - 1) + m^2(1 - 2x_1))} \cdot F_0(x_1) \\ T_2 &= \int dx_1 \frac{m^2}{x_1(1 - x_1)(4m_b^2(x_1 - 1) + m^2(1 - 2x_1))} \cdot F_2(x_1). \end{aligned} \quad (26)$$

In the calculation we have kept the mass of f_2 . The effect of the mass is usually regarded as a correction to the leading twist effect. From the above results we can see that the

amplitude with $\lambda = 2$ is suppressed by $(m/m_b)^2$, although it is a leading twist contribution. At twist-2 two gluonic distribution amplitudes characterize the gluon conversion into f_2 with the helicity $\lambda = 0$ and $\lambda = 2$, respectively. They are defined in the light-cone gauge and are invariant under gauge transformations, which respect the gauge condition in Eq.(15). In other gauges a gauge link between the two gluonic operators in F_0 and in F_2 should be added to make them gauge invariant. Because of the momentum-conservation in the $+$ -direction, the functions $F_0(x)$ and $F_2(x)$ becomes zero if $x > 1$ or $x < 0$. These functions are defined at certain renormalization scale μ , hence they depends not only on x but also on μ . The evolution of these twist-2 operators are well known. Our results given above receive corrections from orders of higher twist and the corrections are suppressed by $(\Lambda/m_b)^2$ where Λ can be Λ_{QCD} or k^- .

With these results in Eq.(25) and Eq.(26) we complete the analysis at twists-2 level. The contributions at twist-3 come from the first and the second term in Eq.(17) and also from contributions with emission of three gluons, which are represented by Fig.2. We will analyse these contributions in the next section and show that the amplitude at twist-3 level is nonzero for $\lambda = 1$.

3. Twist-3 Contributions

In this section we calculate the contribution at order of twist-3. It turns out that at this order the decay amplitude is nonzero only for $\lambda = 1$. We will neglect in this section the effect of m . The effect may be included and including it will results in complicated expressions. We start with Eq.(17). There are two terms which are twist-3 contributions, one term is specified by that one of the indices μ and ν is $-$ in the first term, while another term is specified by the indices μ and ν to be all transversal in the second term. Examining their Lorentz structure we find it is nonzero for $\lambda = \pm 1$. Hence we obtain the leading-twist contribution for $\lambda = 1$ as

$$\begin{aligned}\Gamma_2^{\mu\nu}(k, q_1, 1) &= \Gamma_2^{(1)\mu\nu}(k, q_1) + \Gamma_2^{(2)\mu\nu}(k, q_1) \\ \Gamma_2^{(1)\mu\nu}(k, q_1) &= (2\pi)^3 \delta(q_1^-) \delta^2(q_{1T}) \\ &\quad \cdot \int dx^- e^{-iq_1^+ x^-} \langle f_2(1) | n^\mu G^{a,-}(x^-) G_T^{a,\nu}(0) + n^\nu G_T^{a,\mu}(x^-) G^{a,-}(0) | 0 \rangle \\ \Gamma_2^{(2)\mu\nu}(k, q_1) &= i(2\pi)^3 \delta(q_1^-) \frac{\partial}{\partial q_{1T}^\rho} \delta^2(q_{1T}) \cdot \int dx^- e^{-iq_1^+ x^-} \langle f_2(1) | \partial_T^\rho G_T^{a,\mu}(x^-) G_T^{a,\nu}(0) | 0 \rangle, \quad (27)\end{aligned}$$

where $G_T^{a,\mu} = d_T^{\mu\nu} G_\nu^a$. For $\Gamma_2^{(1)\mu\nu}(k, q_1)$ we can define a distribution amplitude

$$G_0(x) = \frac{1}{2\pi} \int dx^- e^{-ixk^+ x^-} \langle f_2(\lambda) | G_T^{a,+}(x^-) G_T^{a,\mu}(0) | 0 \rangle \omega_\mu(1). \quad (28)$$

It should be noted that G^- in the light cone gauge is not an independent dynamical freedom. Its relation to the gauge field G_T^μ and quark fields can be obtained by solving equation of motion of QCD. One can obtain that G^{+-} can be expressed by $D_T^\mu G_{T\mu}$ and the color-charge current of quarks, where D^μ is the covariant derivative in the adjoint representation of $SU(3)$:

$$[D^\mu]_{ab} = \partial^\mu \delta_{ab} + g_s f_{abc} G^{c,\mu}, \quad \text{for } a, b = 1, \dots, 8. \quad (29)$$

Because of $D_T^\mu G_{T\mu}$ the contribution is a twist-3 contribution. We will not try to give the result for G^- , but leave it with the compact form. Expressing $\Gamma_2^{(1)\mu\nu}(k, q_1)$ with $G_0(x)$ we obtain the contribution to the S -matrix element $\langle \gamma f_2(1) | S | \Upsilon \rangle_0$:

$$\begin{aligned} \Gamma_2^{(1)\mu\nu}(k, q_1) &= \frac{2\pi}{k^+(k-q_1)^+} \{n^\mu \omega^{*\nu} G_0(x_1) + n^\nu \omega^{*\mu} G_0(1-x_1)\}, \\ \langle \gamma f_2(1) | S | \Upsilon \rangle_0 &= \frac{i}{6} e Q_b g_s^2 (2\pi)^4 \delta^4(2p-k-q) \varepsilon^* \cdot \omega^*(1) l^i \langle 0 | \chi^\dagger \sigma^i \psi | \Upsilon \rangle \frac{1}{\sqrt{2m_b^3}} \\ &\quad \cdot \int dx \frac{2}{x(1-x)} G_0(x). \end{aligned} \quad (30)$$

The Lorentz structure of the integral in $\Gamma_2^{(2)\mu\nu}(k, q_1)$ can be decomposed as:

$$\begin{aligned} C^{\mu\nu\rho} &= \int dx^- e^{-iq_1^+ x^-} \langle f_2(1) | \partial_T^\rho G_T^{a,\mu}(x^-) G_T^{a,\nu}(0) | 0 \rangle \\ &= \frac{1}{k^+(q_1-k)^+} \int dx^- e^{-iq_1^+ x^-} \langle f_2(1) | \partial_T^\rho G_T^{a,+\mu}(x^-) G_T^{a,+\nu}(0) | 0 \rangle \\ &= \frac{k^+}{4k^+(q_1-k)^+} \left\{ (\omega^{*\mu} d_T^{\rho\nu} + \omega^{*\nu} d_T^{\rho\mu} - 3\omega^{*\rho} d_T^{\mu\nu}) K_0(x_1) \right. \\ &\quad + \frac{1}{4} (\omega^{*\rho} d_T^{\mu\nu} + \omega^{*\nu} d_T^{\rho\mu} - 3\omega^{*\mu} d_T^{\rho\nu}) K_1(x_1) \\ &\quad \left. + \frac{1}{4} (\omega^{*\rho} d_T^{\mu\nu} + \omega^{*\mu} d_T^{\rho\nu} - 3\omega^{*\nu} d_T^{\rho\mu}) K_2(x_1) \right\}, \end{aligned} \quad (31)$$

where the functions are given by

$$\begin{aligned} K_0(x) &= \frac{1}{k^+} \int dx^- e^{-ixk^+ x^-} \langle f_2(1) | \partial_T^\rho G_T^{a,+\mu}(x^-) G_T^{a,+}_\mu(0) | 0 \rangle \omega_\rho(1), \\ K_1(x) &= \frac{1}{k^+} \int dx^- e^{-ixk^+ x^-} \langle f_2(1) | \partial_T^\mu G_T^{a,+\rho}(x^-) G_T^{a,+}_\mu(0) | 0 \rangle \omega_\rho(1), \\ K_2(x) &= \frac{1}{k^+} \int dx^- e^{-ixk^+ x^-} \langle f_2(1) | \partial_T^\mu G_T^{a,+}_\mu(x^-) G_T^{a,+\rho}(0) | 0 \rangle \omega_\rho(1), \end{aligned} \quad (32)$$

and they have the following property:

$$K_0(x) = -K_0(1-x), \quad K_1(x) = -K_2(1-x). \quad (33)$$

With these results we obtain the contribution to the S -matrix element from $\Gamma_2^{(2)\mu\nu}(k, q_1)$:

$$\begin{aligned} \langle \gamma f_2(1) | S | \Upsilon \rangle_1 &= \frac{i}{6} e Q_b g_s^2 (2\pi)^4 \delta^4(2p-k-q) \varepsilon^* \cdot \omega^*(1) l^i \langle 0 | \chi^\dagger \sigma^i \psi | \Upsilon \rangle \\ &\quad \frac{1}{\sqrt{2m_b^3}} \frac{1}{2\pi} \int dx_1 \frac{1-2x_1}{(1-x_1)^2 x_1^2} K_0(x_1), \\ \langle \gamma f_2(1) | S | \Upsilon \rangle_2 &= \frac{i}{6} e Q_b g_s^2 (2\pi)^4 \delta^4(2p-k-q) \varepsilon^* \cdot \omega^*(1) l^i \langle 0 | \chi^\dagger \sigma^i \psi | \Upsilon \rangle \\ &\quad \frac{1}{\sqrt{2m_b^3}} \frac{1}{2\pi} \int dx_1 \frac{2}{x_1(1-x_1)^2} K_2(x_1). \end{aligned} \quad (34)$$

In the above results $G_0(x)$ is gauge invariant, hence the contribution $\langle \gamma f_2(1) | S | \Upsilon \rangle_0$ also does. But the function $K_0(x)$ and $K_1(x)$ is not gauge invariant, the contributions in Eq.(34) also are not gauge invariant. The gauge invariance can be achieved by considering the emission of three gluons, which we will consider in the below.

The contribution from type of diagrams given in Fig.2a can be written after the expansion in Eq.(7):

$$w_{3a} = \frac{i}{6} e Q_b g_s^3 (2\pi)^4 \delta^4(2p - k - q) \varepsilon_\rho^* \langle 0 | \chi^\dagger \sigma^\ell \psi | \Upsilon \rangle \cdot \frac{i}{4} \cdot \frac{1}{3!} \cdot \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \Gamma_{3a\mu_1\mu_2\mu_3}(k, q_1, q_2, \lambda) \cdot R_{3a}^{\mu_1\mu_2\mu_3\rho\ell}(p, k, q_1, q_2), \quad (35)$$

with

$$\Gamma_{3a}^{\mu_1\mu_2\mu_3}(k, q_1, q_2, \lambda) = \int dx^4 dy^4 e^{-iq_1 \cdot x} e^{-iq_2 \cdot y} f_{abc} \langle f_2(\lambda) | G^{a,\mu_1}(x) G^{b,\mu_2}(y) G^{c,\mu_3}(0) | 0 \rangle. \quad (36)$$

In Eq.(35) the factor $i/4$ comes from the trace over color indices, the three gluons forms a C-even state, hence the contribution is proportional to f_{abc} . The factor $1/3!$ is a statistical factor. The leading order of w_{3a} is twist-3, its contribution is given by keeping the x^- and y^- dependence in the matrix element only and by all gluon fields being transversal. By examining the Lorentz structure of this contribution it is nonzero only for $\lambda = \pm 1$. The leading term of Γ_{3a} for $\lambda = 1$ can be written:

$$\Gamma_{3a}^{\mu_1\mu_2\mu_3}(k, q_1, q_2, 1) = (2\pi)^6 \delta(q_1^-) \delta(q_2^-) \delta^2(q_{1T}) \delta^2(q_{2T}) C_{3a}^{\mu_1\mu_2\mu_3}, \quad (37)$$

where the tensor $C_{3a}^{\mu_1\mu_2\mu_3}$ is given by

$$\begin{aligned} C_{3a}^{\mu_1\mu_2\mu_3} = & \frac{1}{4} (\omega^{*\mu_2} d_T^{\mu_1\mu_3} + \omega^{*\mu_3} d_T^{\mu_1\mu_2} - 3\omega^{*\mu_1} d_T^{\mu_2\mu_3}) H_0(x_1, x_2) \\ & + \frac{1}{4} (\omega^{*\mu_1} d_T^{\mu_2\mu_3} + \omega^{*\mu_3} d_T^{\mu_1\mu_2} - 3\omega^{*\mu_2} d_T^{\mu_1\mu_3}) H_1(x_1, x_2) \\ & + \frac{1}{4} (\omega^{*\mu_1} d_T^{\mu_2\mu_3} + \omega^{*\mu_2} d_T^{\mu_1\mu_3} - 3\omega^{*\mu_3} d_T^{\mu_1\mu_2}) H_2(x_1, x_2). \end{aligned} \quad (38)$$

The functions are defined by:

$$\begin{aligned} H_0(x_1, x_2) &= \int dx^- dy^- e^{-ix_1 k^+ x^- - ix_2 k^+ y^-} f_{abc} \langle f_2(1) | G_T^{a,\mu}(x^-) G^{b,\nu}(y^-) G_\nu^c(0) | 0 \rangle \omega_\mu(1), \\ H_1(x_1, x_2) &= \int dx^- dy^- e^{-ix_1 k^+ x^- - ix_2 k^+ y^-} f_{abc} \langle f_2(1) | G^{a,\nu}(x^-) G_T^{b,\mu}(y^-) G_\nu^c(0) | 0 \rangle \omega_\mu(1), \\ H_3(x_1, x_2) &= \int dx^- dy^- e^{-ix_1 k^+ x^- - ix_2 k^+ y^-} f_{abc} \langle f_2(1) | G^{a,\nu}(x^-) G_\nu^b(y^-) G_T^{c,\mu}(0) | 0 \rangle \omega_\mu(1), \end{aligned} \quad (39)$$

where the variable x_1 and x_2 are related to q_1^+ and q_2^+ by

$$q_1^+ = x_1 k^+ \text{ and } q_2^+ = x_2 k^+, \quad (40)$$

respectively. The functions are related to each other:

$$H_0(x_1, x_2) = -H_1(x_2, x_1), \quad H_2(x_1, x_2) = H_0(1 - x_1 - x_2, x_1), \quad (41)$$

and there are also some identities among them, one of them, which will be used later, is given by

$$H_2(x_1, x_2) = -H_2(x_2, x_1). \quad (42)$$

After a tedious calculation we obtain the leading-twist contribution of Γ_{3a} to the S -matrix element

$$\begin{aligned} \langle \gamma f_2(1) | S | \Upsilon \rangle_3 &= \frac{i}{6} e Q_b g_s^2 (2\pi)^4 \delta^4(2p - k - q) \varepsilon^* \cdot \omega^*(1) l^i \langle 0 | \chi^\dagger \sigma^i \psi | \Upsilon \rangle \\ &\quad \frac{ig_s}{\sqrt{2}m_b^3} \frac{(k^+)^2}{(2\pi)^2} \int dx_1 dx_2 \frac{x_1 + 2x_2 - 1}{x_1 x_2 - x_1 + x_2^2 - x_2} \cdot H_0(x_1, x_2). \end{aligned} \quad (43)$$

We find that this contribution can be combined with $\langle \gamma f_2(1) | S | \Upsilon \rangle_1$ to form a gauge-invariant contribution. For doing this, we insert an identity into $\langle \gamma f_2(1) | S | \Upsilon \rangle_1$:

$$\frac{k^+}{2\pi} \int dx_2 \int dy^- e^{-ix_2 k^+ y^-} = \int dx_2 \delta(x_2) = 1 \quad (44)$$

we obtain

$$\begin{aligned} \langle \gamma f_2(1) | S | \Upsilon \rangle_1 &= (\cdots) \frac{1}{(2\pi)^2} \int dx_1 dx_2 \frac{1 - 2x_1}{(1 - x_1)^2 x_1^2} \\ &\quad \int dx^- dy^- e^{-ix_1 k^+ x^- - ix_2 k^+ y^-} \langle f_2(1) | \partial_T^\rho G_T^{a,+\mu}(x^-) G_T^{a,+}_\mu(0) | 0 \rangle \omega_\rho(1), \end{aligned} \quad (45)$$

where (\cdots) stands for some common factors. For $\langle \gamma f_2(1) | S | \Upsilon \rangle_3$ we make the variable change $x_1 \rightarrow x_2$ and $x_2 \rightarrow x_1$ and write some gluon fields into field strength tensors:

$$\begin{aligned} \langle \gamma f_2(1) | S | \Upsilon \rangle_3 &= (\cdots) \frac{-g_s}{(2\pi)^2} \int dx_1 dx_2 \frac{x_2 + 2x_1 - 1}{x_1(1 - x_1 - x_2)(x_1 x_2 - x_2 + x_1^2 - x_1)} \\ &\quad \int dx^- dy^- e^{-ix_1 k^+ x^- - ix_2 k^+ y^-} f_{abc} \langle f_2(1) | G_T^{a,\mu}(y^-) G^{b,+\nu}(x^-) G^{c,+}_\nu(0) | 0 \rangle \omega_\mu(1), \end{aligned} \quad (46)$$

where (\cdots) stands for the same common factors as in Eq.(45). We note that by setting $x_2 = 0$ the function in the first line of Eq.(46) is the same in the first line of Eq.(45). Because of the identity in Eq.(44) we can combine the two terms together as

$$\begin{aligned} &\langle \gamma f_2(1) | S | \Upsilon \rangle_1 + \langle \gamma f_2(1) | S | \Upsilon \rangle_3 \\ &= (\cdots) \frac{1}{(2\pi)^2} \int dx_1 dx_2 \frac{x_2 + 2x_1 - 1}{x_1(1 - x_1 - x_2)(x_1 x_2 - x_2 + x_1^2 - x_1)} \\ &\quad \int dx^- dy^- e^{-ix_1 k^+ x^- - ix_2 k^+ y^-} \langle f_2(1) | \partial_T^\mu G^{a,+\nu}(x^-) G^{a,+}_\nu(0) \\ &\quad - g_s f_{abc} G_T^{a,\mu}(y^-) G^{b,+\nu}(x^-) G^{c,+}_\nu(0) | 0 \rangle \omega_\mu(1). \end{aligned} \quad (47)$$

We note that the two terms in the matrix element of the above equation can be written as a one term with the covariant derivative $D_T^\mu(y)$ which is defined as

$$[D^\mu(y)]_{ab} = \partial^\mu \delta_{ab} + g_s f_{abc} G^{c,\mu}(y), \quad \text{for } a, b = 1, \dots, 8. \quad (48)$$

With this observation we can define our second distribution amplitude $G_1(x_1, x_2)$:

$$G_1(x_1, x_2) = \frac{1}{(2\pi)^2} \int dx^- dy^- e^{-ix_1 k^+ x^- - ix_2 k^+ y^-} \langle f_2(1) | [D_T^\mu(y^-) G^{+\nu}(x^-)]^a G^{a,+}_{\nu}(0) | 0 \rangle \omega_\mu(1). \quad (49)$$

This amplitude is gauge invariant in the light-cone gauge. In other gauges two gauge links must be supplied to ensure gauge invariance. With it the sum can be written:

$$\begin{aligned} & \langle \gamma f_2(1) | S | \Upsilon \rangle_1 + \langle \gamma f_2(1) | S | \Upsilon \rangle_3 \\ &= (\cdots) \frac{1}{(2\pi)^2} \int dx_1 dx_2 \frac{x_2 + 2x_1 - 1}{x_1(1 - x_1 - x_2)(x_1 x_2 - x_2 + x_1^2 - x_1)} \cdot G_1(x_1, x_2). \end{aligned} \quad (50)$$

Now we turn to the contribution represented by Fig.2b, where the three-gluon vertex is involved. This contribution can be obtained from w_2 in Eq.(9) by using perturbative theory with the three-gluon vertex for Γ_2 once. We denote the term by using the perturbative theory for Γ_2 as Γ_{23} , which is given by:

$$\begin{aligned} \Gamma_{23}^{\mu\nu}(k, q_1, \lambda) &= i g_s f_{abc} \int dx^4 e^{-iq_1 \cdot x} \{ \langle f_2(\lambda) | [i D^{\rho\mu}(q_1) \partial_\sigma G_\rho^a(x) G^{b,\sigma}(x) G^{c,\nu}(0) \\ &\quad - i D^{\sigma\mu}(q_1) \partial_\sigma G_\rho^a(x) G^{b,\rho}(x) G^{c,\nu}(0) + q_{1\sigma} D_\rho^\mu(q_1) G^{a,\sigma}(x) G^{b,\rho}(x) G^{c,\nu}(0)] | 0 \rangle \\ &\quad + (\mu \rightarrow \nu, \nu \rightarrow \mu, q_1 \rightarrow k - q_1) \} \end{aligned} \quad (51)$$

where $D^{\mu\nu}(q_1)$ is the gluon propagator in the light-cone gauge:

$$D^{\mu\nu}(q) = -\frac{1}{q^2} \left(g^{\mu\nu} - \frac{n^\mu q^\nu + n^\nu q^\mu}{n \cdot q} \right). \quad (52)$$

The leading twist of this term is 3, whose contribution comes from the second term in Eq.(51) with $\partial_\sigma = \partial_- = \partial^+$. Again, here the tensor meson can only have $\lambda = \pm 1$. We denote the corresponding contribution to the S -matrix element as $\langle \gamma f_2(1) | S | \Upsilon \rangle_4$, which is then given by

$$\begin{aligned} \langle \gamma f_2(1) | S | \Upsilon \rangle_4 &= i \frac{1}{24} e Q_b g_s^2 (2\pi)^4 \delta^4(2p - k - q) \varepsilon_\rho^* \langle 0 | \chi^\dagger \sigma^\ell \psi | \Upsilon \rangle \\ &\quad \int \frac{d^4 q_1}{(2\pi)^4} R_2^{\mu\nu\rho\ell}(p, k, q_1) \{ (2\pi)^3 \delta(q_1^-) \delta^2(q_{1T}) g_s D^{-\mu}(q_1) \\ &\quad \int dx^- e^{-ik_1^+ x^-} f_{abc} \langle f_2(1) | G^{a,+ \sigma}(x^-) G_\sigma^b(x^-) G^{c,\nu}(0) | 0 \rangle \\ &\quad + (\mu \rightarrow \nu, \nu \rightarrow \mu, q_1 \rightarrow k - q_1) \}. \end{aligned} \quad (53)$$

At the first glance the above contribution may be divergent because of the gluon propagator is singular if we set $q_1^\mu = (q_1^+, 0, 0, 0)$. However we find that the integrand is finite when $q_1^\mu \rightarrow (q_1^+, 0, 0, 0)$, and it leads to a finite contribution which is the same if we replace the gluon propagator by its special propagator $\hat{D}^{\mu\nu}(q_1)$:

$$D^{\mu\nu}(q_1) \rightarrow \hat{D}^{\mu\nu}(q_1) = \frac{n^\mu n^\nu}{(n \cdot q_1)^2}. \quad (54)$$

A discussion of the special propagator can be found in [7], where it is shown that by using the special propagator of quarks twist-4 contributions to the hadron structure functions can be conveniently analysed.

By taking care of the singularity the calculation is straightforward. We obtain

$$\begin{aligned} \langle \gamma f_2(1) | S | \Upsilon \rangle_4 &= \frac{i}{6} e Q_b g_s^2 (2\pi)^4 \delta^4(2p - k - q) \varepsilon^* \cdot \omega^*(1) l^i \langle 0 | \chi^\dagger \sigma^i \psi | \Upsilon \rangle \\ &\quad \frac{ig_s}{\sqrt{2}m_b^3} \int dx_1 \frac{2}{x_1^2(1-x_1)} \cdot J_0(x_1) \end{aligned} \quad (55)$$

with $J_0(x_1)$ defined as

$$J_0(x_1) = \frac{1}{2\pi k^+} \int dx^- e^{-ix_1 k^+ x^-} f_{abc} \langle f_2(1) | G_T^{a,+ \mu}(x^-) G_\mu^b(x^-) G_\nu^{c,+}(0) | 0 \rangle \omega_\nu(1). \quad (56)$$

To ensure the gauge invariance of the S -matrix element, the sum $\langle \gamma f_2(1) | S | \Upsilon \rangle_2 + \langle \gamma f_2(1) | S | \Upsilon \rangle_4$ should be gauge invariant. As the expressions for both terms stand, it does not look like that the sum can be written in a gauge-invariant form. A re-arrangement is needed. We first write $J_0(x_1)$ as an double distribution amplitude by using

$$G_\mu^b(x^-) = \frac{k^+}{2\pi} \int dx_2 dy^- e^{-ix_2 k^+(y-x)^-} G_\mu^b(y^-). \quad (57)$$

Then the contribution can be written as

$$\begin{aligned} \langle \gamma f_2(1) | S | \Upsilon \rangle_4 &= (\cdots) \frac{g_s}{(2\pi)^2} \int dx_1 dx_2 \frac{2}{(x_1 + x_2)^2 (1 - x_1 - x_2)} \\ &\quad \int dx^- dy^- e^{-ix_1 k^+ x^- - ix_2 k^+ y^-} f_{abc} \langle f_2(1) | G_T^{a,+ \mu}(x^-) G_\mu^b(y^-) G_\nu^{c,+}(0) | 0 \rangle \omega_\mu(1), \end{aligned} \quad (58)$$

where (\cdots) stands for the same common factors as before. We add to $\langle \gamma f_2(1) | S | \Upsilon \rangle_4$ a term

$$(\cdots) \frac{g_s}{(2\pi)^2} \int dx_1 dx_2 \frac{2(1 - 2x_1 - 2x_2)}{(x_1 + x_2)(1 - x_1 - x_2)} H_2(x_1, x_2), \quad (59)$$

which is identically zero because of the property in Eq.(42). With this term the contribution can be written:

$$\begin{aligned} \langle \gamma f_2(1) | S | \Upsilon \rangle_4 &= (\cdots) \frac{g_s}{(2\pi)^2} \int dx_1 dx_2 f_2(x_1, x_2) \\ &\quad \int dx^- dy^- e^{-ix_1 k^+ x^- - ix_2 k^+ y^-} f_{abc} \langle f_2(1) | G_T^{a,+ \mu}(x^-) G_\mu^b(y^-) G_\nu^{c,+}(0) | 0 \rangle \omega_\mu(1), \\ f_2(x_1, x_2) &= \left\{ \frac{2}{(x_1 + x_2)^2 (1 - x_1 - x_2)} - \frac{2(1 - 2x_1 - 2x_2)}{x_1(x_1 + x_2)(1 - x_1 - x_2)^2} \right\} \end{aligned} \quad (60)$$

By setting $x_2 = 0$ the function $f_2(x_1, 0)$ is the same as that in the front of $K_2(x_1)$ in Eq. (34). With the same trick used to derive Eq.(47) we can write the sum as

$$\begin{aligned} &\langle \gamma f_2(1) | S | \Upsilon \rangle_2 + \langle \gamma f_2(1) | S | \Upsilon \rangle_4 \\ &= (\cdots) \frac{1}{(2\pi)^2} \int dx_1 dx_2 f_2(x_1, x_2) \\ &\quad \int dx^- dy^- e^{-ix_1 k^+ x^- - ix_2 k^+ y^-} \langle f_2(1) | \partial_T^\nu G^{a,+}_\nu(x^-) G^{a,+ \mu}(0) \\ &\quad + g_s f_{abc} G^{a,+ \nu}(x^-) G_\nu^b(y^-) G_\nu^{c,+ \mu}(0) | 0 \rangle \omega_\mu(1). \end{aligned} \quad (61)$$

Again the two terms in the matrix element can be combined together with the covariant derivative in Eq. (48). We can define the third distribution amplitude $G_2(x_1, x_2)$:

$$G_2(x_1, x_2) = \frac{1}{(2\pi)^2} \int dx^- dy^- e^{-ix_1 k^+ x^- - ix_2 k^+ y^-} \langle f_2(1) | [D_T^\nu(y^-) G_\nu^+(x^-)]^a G^{a,+\mu}(0) | 0 \rangle \omega_\mu(1). \quad (62)$$

The sum can be written:

$$\langle \gamma f_2(1) | S | \Upsilon \rangle_2 + \langle \gamma f_2(1) | S | \Upsilon \rangle_4 = (\cdots) \frac{1}{(2\pi)^2} \int dx_1 dx_2 f_2(x_1, x_2) G_2(x_1, x_2) \quad (63)$$

Adding different contributions together we obtain the S -matrix element for $\lambda = 1$ at twist-3 level:

$$\begin{aligned} \langle \gamma f_2(1) | S | \Upsilon \rangle &= \frac{i}{6} e Q_b g_s^2 (2\pi)^4 \delta^4(2p - k - q) \varepsilon^* \cdot \omega^*(1) l^i \langle 0 | \chi^\dagger \sigma^i \psi | \Upsilon \rangle \frac{1}{m_b^2} \cdot T_1 \\ T_1 &= \frac{1}{\sqrt{2}m_b} \left[\int dx_1 \frac{2}{x_1(1-x_1)} G_0(x_1) + i \int dx_1 dx_2 (f_1(x_1, x_2) G_1(x_1, x_2) \right. \\ &\quad \left. + 2f_2(x_1, x_2) G_2(x_1, x_2)) \right], \end{aligned} \quad (64)$$

where the function f_1 is given by

$$f_1(x_1, x_2) = \frac{x_2 + 2x_1 - 1}{x_1(1-x_1-x_2)(x_1x_2-x_2+x_1^2-x_1)}. \quad (65)$$

With the results given in Eq.(25), Ea.(26) and Eq.(64) we completed the analysis for the decay $\Upsilon \rightarrow \gamma + f_2$, in which the nonperturbative effect related to Υ and that related to f_2 are separated, these effects are parameterized by NRQCD matrix elements and the distribution amplitudes F_0 , F_1 and $G_i (i = 0, 1, 2)$, respectively. The distribution amplitudes characterize how gluons are converted into the tensor meson f_2 . Their definitions are given in this and last section. They are invariant under Lorentz boosts in the z -direction, and are gauge invariant. With the analysis we find that the decay amplitude with $\lambda = 1$ and $\lambda = 2$ are suppressed by the power Λ/m_b and $(\Lambda/m_b)^2$ respectively, in comparison with the decay amplitude with $\lambda = 0$.

4. Comparison with Experiment

In this section we will compare our results with experiment and we will also discuss the radiative decay into η . With the decay amplitudes derived in the last two sections we obtain the decay width:

$$\Gamma(\Upsilon \rightarrow \gamma + f_2) = \frac{2}{9} \pi^2 Q_b^2 \alpha \alpha_s^2(m_b) \left(1 - \frac{m^2}{4m_b^2}\right) \cdot \frac{1}{m_b^4} \langle \Upsilon | O_1^\Upsilon(^3S_1) | \Upsilon \rangle \sum_{\lambda=0}^2 |T_\lambda|^2 \quad (66)$$

where we used

$$\langle \Upsilon | \psi^\dagger \sigma^i \chi | 0 \rangle \langle 0 | \chi^\dagger \sigma^j \psi | \Upsilon \rangle = \langle \Upsilon | O_1^\Upsilon(^3S_1) | \Upsilon \rangle \varepsilon^i (\varepsilon^j)^\dagger. \quad (67)$$

In the above equation ε is the polarization vector of Υ , the matrix element $\langle \Upsilon | O_1^\Upsilon(^3S_1) | \Upsilon \rangle$ is defined in [3] and the average over the spin is implied in the matrix element. As discussed in the last section the relative order of magnitude of the three amplitudes is given by

$$|T_0| : |T_1| : |T_2| = \mathcal{O}(1) : \mathcal{O}\left(\frac{\Lambda}{m_b}\right) : \mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right), \quad (68)$$

where $\Lambda = \Lambda_{QCD}$ or k^- , and each amplitude receives correction at the order $\mathcal{O}(\frac{\Lambda^2}{m_b^2})$ relatively to its leading-order contribution. The corrections are unknown. For consistency we neglect terms with T_1 and T_2 and the mass m of f_2 also should be neglected. Hence we approximate the decay width by

$$\begin{aligned} \Gamma(\Upsilon \rightarrow \gamma + f_2) &= \frac{2}{9} \pi^2 Q_b^2 \alpha \alpha_s^2(m_b) \left(1 - \frac{m^2}{4m_b^2}\right) \cdot \frac{1}{m_b^4} \langle \Upsilon | O_1^\Upsilon(^3S_1) | \Upsilon \rangle |T_0|^2 \\ &\cdot \left\{ 1 + \mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right) + \mathcal{O}(\alpha_s) + \mathcal{O}(v^2) \right\}, \end{aligned} \quad (69)$$

where the orders of possible corrections are given in $\{\dots\}$. For Υ it is a good approximation to neglect these corrections, because m_b is large, while for J/Ψ the corrections may be significant with the relatively small m_c , each correction in $\{\dots\}$ can be at order of 30%. We also neglect these for J/Ψ . To compare with experiment we build the ratio

$$\begin{aligned} R_h(\Upsilon) &= \frac{\Gamma(\Upsilon \rightarrow \gamma + f_2)}{\Gamma(\Upsilon \rightarrow \text{light hadrons})} \\ &\approx \frac{27}{10} Q_b^2 \frac{\alpha}{\alpha_s(m_b)} \frac{\pi^2}{\pi^2 - 9} \left(1 - \frac{m^2}{4m_b^2}\right) \frac{1}{m_b^2} |T_0|^2 \end{aligned} \quad (70)$$

where we used the result at leading orders for $\Gamma(\Upsilon \rightarrow \text{light hadrons})$, where light hadrons are produced through gluon emission. Because Feynman diagrams for $\Gamma(\Upsilon \rightarrow \text{light hadrons})$ have a similar structure as those given in Fig. 1, corrections from higher orders of α_s and of v^2 can be cancelled at certain level in this ratio, we can expect that the ratio receives smaller corrections than the width does. This ratio does not depend on the nonperturbative property of Υ in our approximation. The value of T_0 is unknown, it can be extracted from $R_h(\Upsilon)$, and used to predict $R_h(J/\Psi)$. As an estimation we neglect the μ -dependence of T_0 . With experimental data for hadronic branching ratios we obtain the ratio:

$$\begin{aligned} R &= \frac{B(\Upsilon \rightarrow \gamma + f_2)}{B(J/\Psi \rightarrow \gamma + f_2)} \\ &\approx \frac{0.923}{0.707} \cdot \frac{Q_b^2 m_c^2}{Q_c^2 m_b^2} \cdot \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \cdot \frac{1 - \frac{m^2}{4m_b^2}}{1 - \frac{m^2}{4m_c^2}} \\ &\approx 0.059 \end{aligned} \quad (71)$$

where we have used the parameters:

$$m_c = 1.5\text{GeV}, \quad m_b = 5.0\text{GeV}, \quad \alpha_s(m_c) = 0.3, \quad \alpha_s(m_b) = 0.18. \quad (72)$$

Experimentally, the branching ratio has been measured recently, its value is [2]:

$$B(\Upsilon \rightarrow \gamma + f_2) = (8.1 \pm 2.3_{-2.7}^{+2.9}) \times 10^{-5}. \quad (73)$$

With the value for $B(J/\Psi \rightarrow \gamma + f_2) = 1.38 \times 10^{-3}$ [12] we obtain the ratio determined by experiment:

$$R \approx 0.058 \pm 0.016_{-0.020}^{+0.021}. \quad (74)$$

This value is in agreement with our prediction. However, it should be kept in mind that experimental errors are large and corrections to our prediction can be substantial. Another uncertainty is from the pole-mass of c -quark, which value is not well known as the pole-mass of b -quark. A recent QCD sum rule analysis for charmonium systems [13] shows that m_c can be 1.7GeV. If we vary the mass m_c from 1.3GeV to 1.7GeV, then the ratio R in Eq.(71) changes from 0.044 to 0.076.

As discussed before, we should neglect the decay amplitude with $\lambda = \pm 1$ and with $\lambda = \pm 2$ to consistently predict decay widths in our approximation. However the helicity amplitudes can be measured by measuring the polarization of f_2 , in which the main decay mode $f_2 \rightarrow \pi\pi$ can be used. This type of measurements have been done for J/Ψ -decay [1]. But, as pointed out in [4], the fitting formula used in the measurement for the angular distribution of γ and π was not correct. Hence we still do not know about various helicity amplitudes. With the sample of 5×10^7 J/ψ 's, whose collection will be ended this year at BES, a new analysis will provide such information. In our approach, the angular distribution will be approximated by

$$\begin{aligned} \frac{d\mathcal{N}}{d\cos\theta_\gamma d\phi_\pi d\cos\theta_\pi} &\propto \{(1 + \cos^2\theta_\gamma)(3\cos^2\theta_\pi - 1)^2|T_0|^2 \\ &\quad + \sqrt{3}\sin 2\theta_\gamma \cos\theta_\pi \sin 2\theta_\pi (3\cos^2\theta_\pi - 1)\text{Re}(T_0 T_1^*)\} \\ &\quad \cdot \{1 + \mathcal{O}(\frac{\Lambda^2}{m_b^2})\} \end{aligned} \quad (75)$$

where θ_γ is the polar angle between γ and the e^+e^- -beam axis, θ_π and ϕ_π are the polar and azimuthal angles of a pion in the f_2 -rest frame with respect to the γ -direction, and $\phi_\pi = 0$ is defined by the e^+e^- -beam axis. A general formula for the distribution is provided in [4].

With the experimental result in Eq.(73) one can estimate a phenomenological constant for the gluon conversion into f_2 . For this we write the distribution amplitude as

$$F_0(x) = f_g^S \cdot f(x), \quad \text{with} \quad \int_0^1 f(x) = 1 \quad (76)$$

where the constant f_g^S has dimension 1 in mass and is defined by [5]

$$\begin{aligned} \langle f_2(k) | G_{\alpha\beta}^a(0) G_{\mu\nu}^a(0) | 0 \rangle &= f_g^T \left\{ [(k_\alpha k_\mu - \frac{1}{2}m^2 g_{\alpha\mu})\varepsilon_{\beta\nu}^* - (\alpha \leftrightarrow \beta)] - (\mu \leftrightarrow \nu) \right\} \\ &\quad + \frac{1}{2}f_g^S m^2 \left\{ [g_{\alpha\mu}\varepsilon_{\beta\nu}^* - (\alpha \leftrightarrow \beta)] - (\mu \leftrightarrow \nu) \right\}. \end{aligned} \quad (77)$$

If we take the renormalization scale $\mu = m_b$ as a very large scale, we may take the asymptotic form for the function $f(x)$ at $\mu = m_b$:

$$f(x) = 15x^2(1-x)^2. \quad (78)$$

with the experimental data we obtain the estimation at $\mu = m_b$:

$$f_g^S \approx 44\text{MeV}. \quad (79)$$

Before ending this section we briefly discussed the decay

$$\Upsilon \rightarrow \gamma(q) + \eta(k) \quad (80)$$

This decay has been studied in [4], based on a static quark model for η , and the corresponding decay of J/ψ also has been studied with various approaches [8–11]. In these approaches the decay is controlled by the $U_A(1)$ anomaly, in which the conversion of gluons into η is characterized by a local operator of gluons. In our approach the corresponding operator becomes non-local. Following the results presented in Sect. 2 we can obtain the decay amplitude at twist-2 level:

$$\begin{aligned} \langle \gamma \eta | S | \Upsilon \rangle &= \frac{-i}{48} e Q_b g_s^2 (2\pi)^4 \delta^4(2p - k - q) \varepsilon_\rho \langle 0 | \chi^\dagger \sigma^\ell \psi | \Upsilon \rangle e^{\ell\rho} \frac{1}{m_b^2} \\ &\quad \cdot \frac{m_\eta^2}{m_b^2} \int dx_1 \frac{1 - 2x_1}{x_1(1 - x_1)^2} F_\eta(x_1), \end{aligned} \quad (81)$$

where the tensor $e^{\mu\nu}$ is defined by

$$e^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} l_\alpha n_\beta, \quad (82)$$

where $\varepsilon^{\mu\nu\alpha\beta}$ is totally anti-symmetric with $\varepsilon^{0123} = 1$. The distribution amplitude $F_\eta(x_1)$ characterizes the conversion of two gluons into η , which is defined in the light-cone gauge as

$$F_\eta(x_1) = \frac{1}{2\pi k^+} \int dx^- e^{-ix_1 k^+ x^-} \langle \eta(k) | G^{a,+ \mu}(x^-) G^{a,+ \nu}(0) | 0 \rangle e_{\mu\nu}. \quad (83)$$

Our result shows that the twist-2 contribution is suppressed by m_η^2 . In the twist-expansion the light hadron mass m_η should be taken as a small scale as Λ_{QCD} , hence the contribution is proportional to Λ_{QCD}^2 . This implies that a complete analysis should include not only this contribution but also twist-4 contributions, in which one needs to consider the contributions from emission of 2, 3 and 4 gluons. However, without a complete analysis we can always write the result of a complete analysis as

$$\langle \gamma \eta | S | \Upsilon \rangle = \frac{-i}{48} e Q_b g_s^2 (2\pi)^4 \delta^4(2p - k - q) \varepsilon_\rho \langle 0 | \chi^\dagger \sigma^\ell \psi | \Upsilon \rangle e^{\ell\rho} \frac{1}{m_b^4} \cdot g_\eta, \quad (84)$$

where the parameter g_η has a dimension 3 in mass. The parameter characterized the conversion of gluons into η and it does not depend on properties of Υ . Hence the above result also applies for the J/Ψ decay. Following the above analysis for f_2 we obtain for η :

$$\begin{aligned}
R &= \frac{B(\Upsilon \rightarrow \gamma + \eta)}{B(J/\Psi \rightarrow \gamma + \eta)} \\
&\approx \frac{0.923}{0.707} \cdot \frac{Q_b^2 m_c^6}{Q_c^2 m_b^6} \cdot \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \cdot \frac{1 - \frac{m_\eta^2}{4m_b^2}}{1 - \frac{m_\eta^2}{4m_c^2}} \\
&\approx 4.1 \times 10^{-4}.
\end{aligned} \tag{85}$$

With the experimental data for $B(J/\Psi \rightarrow \gamma + \eta)$ we obtain

$$B(\Upsilon \rightarrow \gamma + \eta) \approx 3.5 \times 10^{-7}. \tag{86}$$

The predicted $B(\Upsilon \rightarrow \gamma + \eta)$ may be too small to be observed.

5. Conclusion

In this work we have studied the radiative decay of a 3S_1 quarkonium into a tensor meson f_2 . A factorization at tree-level is performed for the decay amplitude, in which nonperturbative effect related to the quarkonium and that related to f_2 is separated. These effects are parameterized with NRQCD matrix elements and with a set of gluonic distribution amplitudes, which are defined in this work. These amplitudes are gauge invariant and universal. At twist-2 level, we find two amplitudes characterizing the conversion of two gluons into f_2 with the helicity $\lambda = 0$ and $\lambda = 2$, respectively, and the contribution to the decay amplitude with $\lambda = 2$ is suppressed by m^2 . At twist-3 level the decay amplitude is nonzero only for $\lambda = 1$, three gluonic distribution amplitudes are introduced to describe the gluon conversion. At loops-level, the factorization may still be performed as shown in studies of other exclusive processes [14], but we will not study this subject in this work and leave it for future.

In our approach an expansion in v , the velocity of the b -quark inside Υ in its rest-frame, is used. We have only taken the leading order contributions at v^0 . At this order, Υ can be considered as a bound state of the $b\bar{b}$ pair in a color singlet. The corrections to the leading-order results may also added. However, problems arise at order of v^4 . At this order Υ has a component in which the $b\bar{b}$ pair is in color octet and the component is a bound state of the $b\bar{b}$ pair with soft gluons. It is unclear how to add corrections from this component at order of v^4 . It deserves a further study of these problems to understand a bound state of many dynamical freedoms of QCD.

The NRQCD matrix elements are known from other experiment and form lattice QCD calculations, while there is not many information about the 5 gluonic distribution amplitudes. Using experiment data and our result we extract a parameter relevant for the $\lambda = 0$ amplitude. It should be emphasized that these distribution amplitudes are universal and can be used for production of f_2 in other processes, like the one studied in [5]. In experiment, a measurement of the polarization of f_2 can provide information about these distribution amplitudes with different λ . With the sample of 5×10^7 J/ψ 's, whose collection will be ended this year at BES, such measurement is planed [15].

With our results we can predict the ratio of the branching ratios for Υ and for J/Ψ . An agreement between the prediction and experiment can be obtained. In the approach used

here, we can also perform an analysis for the radiative decay into η . It turns out that twist-4 effect should be included in a complete analysis. But without the complete analysis we are still able to predict the branching ratio for Υ and it may be too small to be observed.

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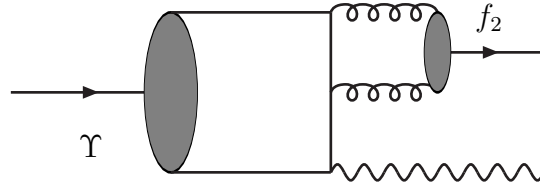


Fig.1

Fig.1: One of the diagrams for contributions of two-gluon emission.

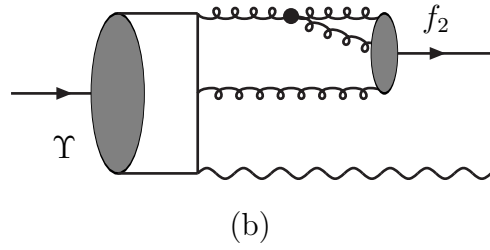
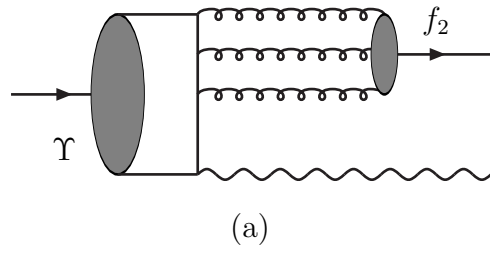


Fig.2

Fig.2: Typical diagrams for contributions with three-gluon emission.